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B.Sc. Part II Paper IV

Formation & Solution of Diff. Eqn.

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Formation of ordinary Differential Equation

We illustrate the method by some simple examples

(i) Let $y^2 = 4ax$ be an eqn. containing x and y and one arbitrary constant a.

Diff. w.r.t. x, we get

$2y\frac{dy}{dx} = 4a$ putting this in $y^2 = 4ax$, we get

$\Rightarrow \quad y^2 = \left(2y\frac{dy}{dx}\right)x$

$\Rightarrow \quad 2x\frac{dy}{dx} = y$ which is a diff. eqn. of 1st order

(ii) Let $x^2+y^2 = 2ax+c$ —— ①

be an eqn. containing x, y and two arbitrary constants a and c.

Differentiating ① w.r.t. x, we have

$2x + 2y\frac{dy}{dx} = 2a \Rightarrow x + y\frac{dy}{dx} = a$

Differentiating again w.r.t. x, we get

$1 + y\frac{d^2y}{dx} + \frac{dy}{dx}\cdot\frac{dy}{dx} = 0$

$\Rightarrow \quad 1 + \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$

which is a diff eqn. of second order and first degree

* In general, if we have an eqn. containing n arbitrary constants, then by differentiating it n times we shall get (n+1) eqns altogether. If we eliminate the n constants from the (n+1) eqns, we shall get a diff. eqn. of the nth order. i.e an eqn. of the form

$$\phi\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \cdots \frac{d^ny}{dx^n}\right) = 0.$$

Solution of a Diff. Eqn. :- In previous we have learnt about obtain a diff. eqn, by the help of successive differentiation or by the process of elimination. Now hereafter we shall study the following reverse process. Given a diff. eqn. we shall strive to obtain an eqn. in x and y which satisfies the given diff. eqn. i.e. from which the given differential equation is derived. This is called a solution of the diff. eqn. Hence we have the following definition:

Any relation connecting the variables of an eqn. and not involving the differential coefficients, such that this relation and the derivatives obtained from it satisfy the given differential equation is called a solution of the diff. eqn.

It will be assumed that an ordinary eqn. of order n has a solution involving n independent arbitrary constants.

Type of Solutions :-

(i) General Solution :- The solution contains a number of arbitrary independent constants equal to the order of the diff. eqn. is called the general solution of the equation, or the complete primitive of the equation.

(ii) Particular Solution :- The solution obtained from the general solution by assigning particular values to one or ~~the complete~~ more of the arbitrary constants is called particular solution.

(iii) Singular solution :- Sometimes it happens that a solution of a diff. eqn. can be found which is distinct from the general and does not contain an arbitrary constant. This is known as singular solution. For example

$$y = x\frac{dy}{dx} + \frac{a}{dy/dx}$$

we shall see that the general solution of this is $y = mx + \frac{a}{m}$ where m is an arbitrary constant. Giving to m any particular value, we shall get a particular solution. Also $y^2 = 4ax$ is a solution (without arbitrary)